Modelling of a Structure under Permanent and Fire Design Situation

Milan HOLICKY
Research fellow
Czech Technical Univ.,
Prague, Czech Rep.

Milan Holicky, born 1943, received his Civil Engineering degree from the Czech Technical University (CTU) in Prague in 1965, Ph.D. from the University of Waterloo in 1971; researcher at the Klokner Institute of CTU.

Jean-Baptiste SCHLEICH
Professor, Principal Engineer
Profil ARBED-Recherches
Esch/Alzette, Luxembourg

Jean-Baptiste Schleich, born 1942, received his Civil Engineering degree from the University of Liege in 1967;
Professor at the University of Liège, since 1998 Convenor of Fire Parts for Eurocodes EC1 & EC4.

Summary

Modelling of a structure under permanent and fire design situation is based on the law of the total probability, proportion of the permanent and fire design situation and conditional probabilities of structural collapse given a particular design situation. Reliability of the structure under both design situations is analysed using probabilistic methods. It is shown that determined conditional probabilities can be used as input data in a developed Bayesian causal network, which seems to provide an effective tool to estimate expected total risk.

Keywords: Permanent and fire design situation, conditional probabilities, Bayesian network, estimation of risk.

1. Introduction

Safety in case of fire is one of the essential requirements imposed on construction works by Council Directive 89/106/EEC [1] and new European documents [2,3] and International Standards [4,5]. Experience and available data [6] to [12] indicate that depending on particular conditions and applied fire protection system, the probability of fire flashover (outbreak) may be expected within a broad range. It appears [13,14,15,16] that probabilistic concepts including Bayesian belief networks provide an effective tool to make estimation of risk in a rational way.

2. Probabilistic Concepts

Probabilistic approach to verification of structural reliability and safety should consider all possible events that might lead to unfavourable effects. These events are often caused by accidental actions as fire, impact, explosion, and extreme climatic loads. In the following it is assumed that adequate situations H_i (based on hazard scenarios and common design situations) occur during the specified design period with the probability $\mathrm{P}\{H_i\}$. If the failure (unfavourable consequence) of the structure F due to a particular situation H_i occurs with the conditional probability $\mathrm{P}\{F|H_i\}$, then the total probability of failure p_F is given by the law of total probability [17] as:

$$p_F = \sum \mathrm{P}\{F|H_i\}\,\mathrm{P}\{H_i\} \qquad (1)$$

Equation (1) can be used to harmonise partial probabilities $\mathrm{P}\{F|H_i\}\,\mathrm{P}\{H_i\}$ in order to comply with the design condition $p_F < p_\mathrm{t}$, where p_t is a target (design) probability of failure. The target value p_t may be determined using probabilistic optimisation, however, up to now it is mostly based on a past experience (e.g. $7{,}23 \times 10^{-5}$ per 55 years considered in [2]).

In the presented study two basic design situations for a given structure are considered only:

- H_1 permanent design situation, assumed to occur with the probability $\mathrm{P}\{H_1\}=0{,}9$;
- H_2 accidental design situation due to fire starting, assumed to occur with the probability $\mathrm{P}\{H_2\} = 0{,}1$ (corresponds to an office area of 250 m^2 , [16]).

The accidental design situation H_2 may lead to another two subsequent situations:

- H_3 accidental design situation without fire flashover (which is assumed to occur with the probability $P\{H_3|H_2\}$ = 0,934);
- H_4 accidental design situation with fire flashover (which is assumed to occur with the probability $P\{H_4|H_2\}$ = 0,066).

The conditional probabilities $P\{H_3|H_2\}$ and $P\{H_4|H_2\}$ indicated above were obtained in previous studies [15,16] for a structure without sprinklers; with sprinklers these probabilities are 0,998 and 0,002 respectively. Considering the above-defined situations it follows from general equation (1) that the total probability of failure p_F can be written as

$$p_F = P\{F|H_1\}\ P\{H_1\} + \left[P\{F|H_3\}\ P\{H_3|\ H_2\} + P\{F|H_4\}\ P\{H_4|\ H_2\}\right] P\{H_2\} \tag{2}$$

The conditional probabilities $P\{F|H_i\}$ entering equation (1) must be determined by a separate probabilistic analysis of the respective situations H_i. The traditional reliability methods [18,19,20,21, 22] assume, that the structural failure F is unambiguously described by inequality $g(\boldsymbol{x}) < 0$, where $g(\boldsymbol{x})$ is the so called limit state ("performance") function defined in the domain of the vector of basic variables $\boldsymbol{X}$, where $\boldsymbol{x}$ is a realisation of $\boldsymbol{X}$. Note that $g(\boldsymbol{x}) = 0$ describes the limit state and inequality $g(\boldsymbol{x}) > 0$ the safe state of a structure. If $f_{\boldsymbol{X}}(\boldsymbol{x}|H_i)$ indicates joint probability density of basic variables $\boldsymbol{X}$ given situation H_i, the conditional probability $P\{F|H_i\}$ can be determined [4] as

$$P\{F|H_i\} = \int_{g_i(\boldsymbol{x})<0} f_{\boldsymbol{X}}(\boldsymbol{x} \mid H_i)\mathrm{d}\boldsymbol{x} \tag{3}$$

It should be mentioned that the probability $P\{F|H_i\}$ calculated using equation (2) may be affected by two essential deficiencies [21]: uncertainty in the definition of the limit state function $g_i(\boldsymbol{x})$, and uncertainty in the theoretical model $f_{\boldsymbol{X}}(\boldsymbol{x}|H_i)$ of basic variables $\boldsymbol{X}$.

The total probability of failure p_F given by (1) can be used for a probabilistic analysis with respect to a chosen decision parameter δ (for example degree of protection against fire flashover). If p_F is dependent on a parameter δ then the objective function in terms of the total cost C can be written as

$$C = C_0 + C_{\mathrm{m}}\delta + p_F(\delta)C_F \tag{4}$$

where C_0 denotes the initial cost, C_{m} the marginal cost and C_F the cost due to structural failure (malfunctioning cost).

3. An example of a steel beam

As an example consider a simply supported steel beam of the span L = 6 m and width of loading area d = 3 m supporting a floor of an office area. The beam is designed in accordance with Eurocodes [2, 3, 4] assuming permanent load G_{k} = 12kN/m (load factor 1,35) and imposed load Q_{k} =3 kN/m^2 (load factor 1,5). The minimum required resistance is $R_{\mathrm{k}} = W f_{\mathrm{yk}} = 625{,}6\times235\times10^{-3} = 147$ kNm, where W is the section modulus and f_{yk} the characteristic strength of steel S235.

Reliability analysis of a structure under both design situations (permanent and fire) is based on specification of limit state functions $g_1(\boldsymbol{x})$, $g_3(\boldsymbol{x})$ and $g_4(\boldsymbol{x})$ corresponding to the situation H_1, H_3 and H_4. The limit state functions $g_1(\boldsymbol{x})$, $g_3(\boldsymbol{x})$ and $g_4(\boldsymbol{x})$ may be written as

$$g_1(\boldsymbol{x}) = \xi_R\, R - \xi_E\,(G + (Q_{\mathrm{S}}+Q_{\mathrm{l}})d)\, L^2/8 \tag{5}$$
$$g_3(\boldsymbol{x}) = \xi_R\, R - \xi_E\,(G + Q_{\mathrm{l}}d)\, L^2/8 \tag{6}$$
$$g_4(\boldsymbol{x}) = \xi_{Rf}\, R\, k_{\mathrm{y}} - \xi_E\,(G + Q_{\mathrm{l}}d)\, L^2/8 \tag{7}$$

where ξ_R, ξ_E, and ξ_{Rf} denote coefficients of model uncertainty. The load effect is given as the bending moment at midspan point due to permanent load G, short term imposed load Q_{s} and long term imposed load Q_{l} (all symbols used here are defined in *Table 1*). When fire is fully developed (fire flashover) the reduction factor k_{y} is dependent on the temperature θ of the unprotected structure [4]

$$k_{\mathrm{y}} = (0{,}9674\ ((1+\exp((\theta - 482)/39{,}19))))^{(-1/3{,}833)} \tag{8}$$
$$\theta = 20 + 1325(1-0{,}324\exp(-0{,}2t\Gamma/60)-0{,}204\exp(-1{,}7t\Gamma/60)-0{,}472\exp(-19t\Gamma/60)) \tag{9}$$

where further t = 0,00013 $q_{t,d}$ Γ / O and the factor $\Gamma = (O/b)^2/(0{,}04/1160)^2$. The theoretical models for all the basic variables are specified in *Table 1*. The fire load intensity $q_{f,d}$ was recalculated assuming that the beam just satisfies design conditions under accidental (fire) design situation.

Table 1. Review of basic variables

Basic variable X	Distribution	Characteristic value X_k	Design value for fire	Mean μ_X	Standard deviation σ_X
$R = W f_y$ - resistance [kNm]	Lognormal	294 = 1251,2 $\times 235 \times 10^{-3}$ (5%)	294	350,3=1251,2 $\times 280 \times 10^{-3}$	$0{,}1 \times \mu_X$ 2)
q_f - fire load [MJ/m^3]	Gumbel	192,7 (80%)	192,7	158,5	$0{,}3 \times \mu_X$
$b=(\rho\, c\lambda)^{0,5}$ [J/m^2 s$^{1/2}$ K]	sLognormal 1)	1160 (50%)	1160	1160 4)	$0{,}05 \times \mu_X$
$O = A_v \sqrt{h}/A_t$ [m$^{1/2}$]	Lognormal	0,04 m$^{1/2}$ (50%)	0,04	0,04 4)	$0{,}1 \times \mu_X$ 3)
$A_{tf} = A_t / A_f$	Constant	4	4	4	0
G - perman. load [kN/m]	Normal	12 (50%)	12	12	$0{,}1 \times \mu_X$
Q_{long} - long term Q [kN/m]	Gumbel	6,75 = 2,25 $\times$ 3 (99,9%)	3,375	1,8 = 0,6$\times$3	$0{,}5 \times \mu_X$
Q_{short} - short term Q [kN/m]	Gumbel	2,25 = 0,75 $\times$ 3 (99,9%)	1,125	0,6 = 0,2$\times$3	$0{,}9 \times \mu_X$
ξ_R - uncert. of R at 20°C	Normal	1 (50%)	1	1,0	0,1
ξ_{Rf} - uncert. of R in fire	Normal	1 (50%)	1	1,0	0,15
ξ_E - uncertainty of E	Normal	1 (50%)	1	1,0	0,1

Notes: 1) The lower bound of shifted Lognormal distribution for b is 1000 J/m^2 s$^{1/2}$ K, which is the value recommended in [3] as the minimum value to be considered in design.
2) Recent research of available data concerning yield strength and geometrical data has indicated that the standard deviation could be less than $0{,}1 \times \mu_X$; this value is considered here as a safe assumption.
3) Opening factor is usually well known except the case when window glass brakes during the fire; then σ_X may vary considerably (this is, however, not considered here).
4) For the given parameters the temperature curve (9) is equivalent to the ISO curve.

Using these models and the above defined limit state functions $g_1(\boldsymbol{x})$, $g_3(\boldsymbol{x})$ and $g_4(\boldsymbol{x})$ the reliability analysis of the unprotected beam has been done using the software product COMREL. The resulting conditional probabilities P{$F|H_1$}, P{$F|H_3$}, P{$F|H_4$} given by equation (3) and the total probability of failure p_F given by equation (2) are

$$\mathrm{P}\{F|H_1\}= 0{,}131 \times 10^{-5} \,;\ \mathrm{P}\{F|H_3\}= 0{,}0485 \times 10^{-5} \,;\ \mathrm{P}\{F|H_4\}= 0{,}486 \,;\ p_F = 0.00321 \tag{10}$$

The total probability of failure p_F = 0.00321 is much greater than $7{,}23 \times 10^{-5}$, and can not be accepted. It should be mentioned, however, that the beam considered in this study is assumed to have the minimum required resistance R, which will hardly ever be used in practice (due to serviceability requirements and standardisation of steel profiles). Furthermore, it is assumed that the beam is not protected and that the heating corresponds to the ISO curve.

One of possible measures that can be considered to improve the fire safety, is improvement of the fire protection system that would lead to a decrease of the conditional probability P{$H_4|H_2$}. Assume that the improvement can be measured by the factor δ ,by which the probability P{$H_4|H_2$} is decreased such that the new proportion of fully developed fires is given by the fraction P{$H_4|H_2$}/δ. *Fig. 1* shows the resulting probabilities P{$H_4|H_2$} and p_F as a function of δ. Further, if the cost necessary to reach δ is $C_m\delta$,then the total cost C is given by equation (4); *Fig. 2* shows the relative value of the total cost C/C_m as a function of δ , and assuming C_0=0 .

It follows that in order to decrease the total probability p_F below the required level $7{,}23 \times 10^{-5}$ (with reliability index 3,8) the factor δ (*see Fig.1)should* be grater than 50. *Fig.2* indicates the optimum value of δ, for example for C_F/C_m = 100 000 units the minimum cost correspond to $\delta \approx 28$. The other possible measures may include reliability increase of other components of fire protection system and application of structural protection.

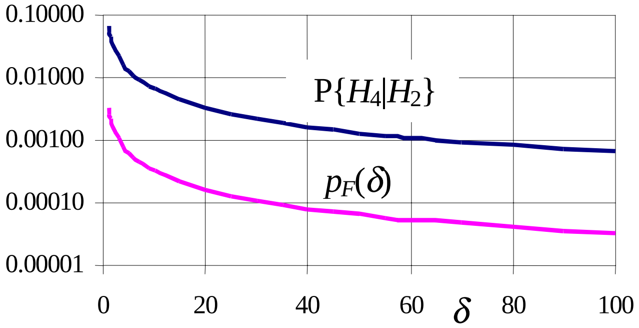

Fig. 1 Probability $P\{H_4|H_2\}$ and $p_F(\delta)$

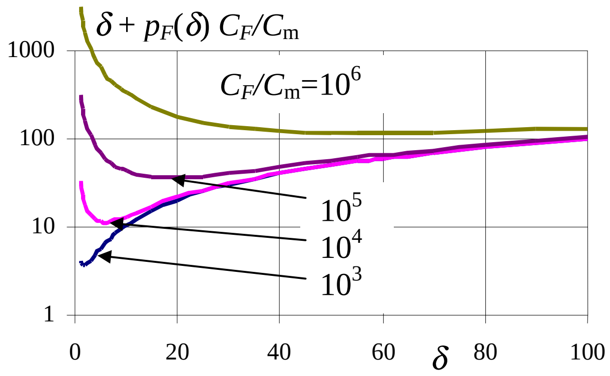

Fig. 2 The relative cost $\delta + p_F(\delta)\, C_F/C_m$

4. Estimation of risk

In general, the situations H_i may cause a number of events E_{ij} (including failure F) having economic consequences (e.g. excessive deformations, full development of the fire). It is assumed that adverse consequences of these events can be normally expressed by one-component quantity C_{ij}. If there is one-to-one mapping between the consequences C_{ij} and the events E_{ij}, then the total risk R related to the considered situations H_i is the sum

$$R = \sum C_{ij}\, P\{E_{ij}|H_i\} P\{H_i\} \tag{11}$$

Depending on particular conditions the total risk R may be generally different from the total cost C given by (4). In some cases it is necessary to describe the consequences of events E_{ij} by the quantity having more components, (for example by the cost, injuries or casualties). Furthermore, the dependence of consequences on events may be more complicated than one-to-one mapping [15,16]. An effective tool to estimate the total risk is the network shown in *Fig.3*, which is a simplified influence diagram described in detail in [15,16].

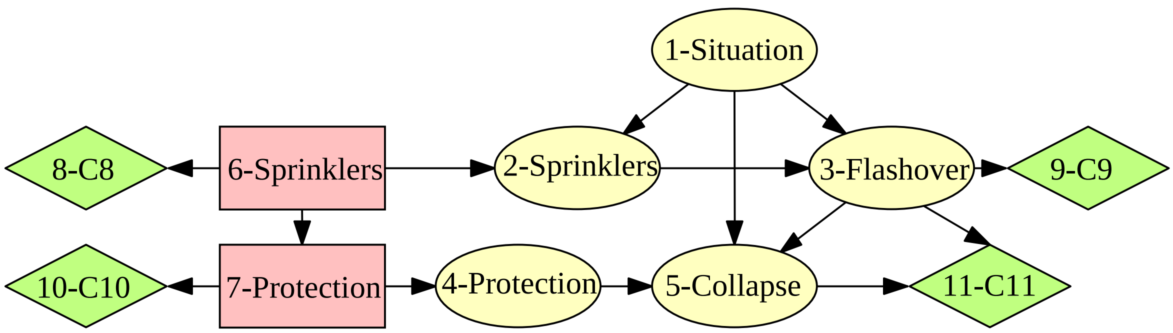

Fig. 3 Network describing a structure under permanent and fire design situation

The network in *Fig. 3* consists of five chance nodes (representing alternative random variable having two states only), numbered 1, 2, 3, 4, and 5, two decision nodes 6 and 7, and four utility (cost) nodes 8, 9, 10, and 11. The chance node 2-Sprinklers describes functioning of sprinklers provided that the decision (node 6) is positive; the probability of active state of sprinklers given fire start is assumed to be very high 0,999. The chance node 3-Flashover represents design situation H_3 (fire design situation without flashover) and H_4 (fire design situation with flashover). Conditional probabilities for the chance node and 5 - Collapse are given in *Table 2*. If the sprinklers are installed, the flashover (chance node 3) has the positive state with the conditional probability 0,002

[16]; if the sprinklers are not installed then P{H_4/H_2} = 0,066 [15]. The chance node 4-Protection depends of the decision represented by the node 7. Finally the chance node 5-Collapse represents structural failure described by the probability distribution given in *Table 2* (see equation (10)).

Table 2. Conditional probabilities of structural collapse (node 5 - Collapse)

1 - Situation	Permanent		Fire (ISO curve is considered)			
2 - Flashover	No		Yes		No	
3 - Protection	Yes	No	Yes	No	Yes	No
5- Collapse yes	0,00000131		0,01	0,486	0,000000485	
5- Collapse no	0,99999869		0,99	0,514	0,999999515	

Further it is assumed that the cost due to flashover C_9 is 500 units, cost of sprinklers C_8 is 10 units, cost of structural protection C_{10} is 60 units and cost due to structural failure C_{11} is considered within the interval from 10^3 to 10^6 units. Under these assumptions the failure probability p_F and the total expected risk R given by equation (11) and obtained using the programme HUGIN [23], are shown in *Table 3* and *Fig. 4*, for four possible decisions concerning sprinklers and structural protection.

Table 3. Probabilities of failure p_F and risks R

Sprinklers	Yes		No	
Protection	Yes	No	Yes	No
Decision	1	2	3	4
$p_F \times 10^5$	0,33	10,15	6,72	321
Cost C_{11}	Risk R			
1000	70,1	10,2	63,4	6,5
10000	70,1	11,1	64,0	35,4
100000	70,4	20,3	70,0	324
1000000	73,4	111	130	3212

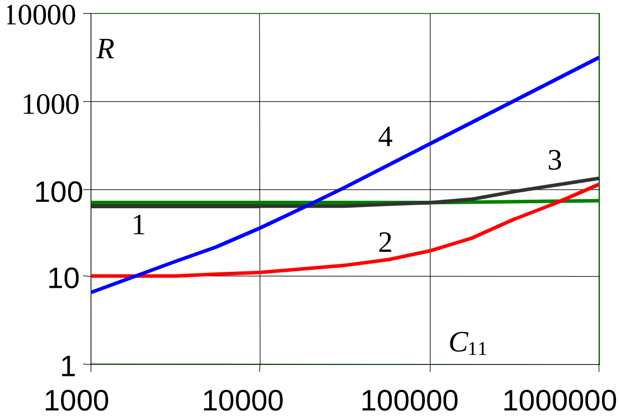

Fig. 4 The total risk R for decision 1, 2, 3, and 4

It follows from *Table 3* and *Fig. 4* that decision 2 - sprinklers without protection - leads to the minimum risk R for a broad range of the cost C_{11} (from about 2200 to about 650000 units). For the cost C_{11} less than 2200 the decision 4 - neither sprinklers nor protection- appears to be more economic, though it has obviously the greatest total probability of failure $p_F = 321 \times 10^{-5}$ (see *Table 3*). For the cost C_{11} greater than about 650000 units, the minimum risk seems to be provided by decision 1, when both sprinklers and structural protection are applied.

5. Conclusions

Probabilistic concepts proof to provide basic concepts for analysing structural reliability and safety under various design situations including persistent and accidental design situation due to fire.

Bayesian belief network seems to provide logical, well-defined effective tool to analyse the probability of fire flashover and probability of structural failure for different decisions concerning installation of sprinklers and application of structural protection.

The target probability of failure $7{,}23\times10^{-5}$ (reliability index 3,8) is very likely to be exceeded if neither sprinklers nor structural protection are used.

Bayesian network supplemented by decision and utility nodes (influence diagram) enables to minimise the expected risk under persistent and fire design situations. It appears that for a middle cost C_{11} (less than 650000 units) the decision 2-sprinklers without protection- leads to the lowest expected risk. In this case the total failure probability could be decreased to $p_F = 2{,}19\text{x}10^{-5}$, if the probability of structural collapse due to fire flashover drops for natural fires from 0,486 to 0,1.

Further comprehensive studies focussed on effects of various fire protection measures on the probabilities of fire occurrence and flashover including economic assessment is needed.

6. References

[1] Council Directive (89/106/EEC). *Official Journal of the Eur. Communities*, No 1. 40/13, 1989.
[2] ENV 1991-1:*Basis of Design and Action on Structures–Part1: Basis of Design.* Brussels, 1994.
[3] ENV 1991-2-2: *Basis of Design and Action on Structures – Part 2/2: Actions on Structures - Actions on Structures Exposed to Fire*. Brussels, 1995.
[4] ISO 2394: *General Principles on Reliability for Structures.* Zurich, 1997.
[5] ISO 834: *Fire Resistance - General Requirements.* Zurich, 1995.
[6] BSI DD 240: *Fire Safety Engineering in Building. Part 1: Guide to the Application of Fire Safety Engineering Principles.* London, 1997.
[7] CIB Report, *Action on Structures - Fire,* Publication 166, 1993.
[8] DIN 18230-1: *Baulicher Brandschutz im Industriebau, Teil 1" Rechnerisch erforderliche Feuerwiderstandsdauer*. Berlin, 1998.
[9] ECCS *Working Document No. 94*: *Background document to Eurocode 1*. 1996.
[10] Schleich J.-B.,Cajot L.-G. :ECSC Fin. Rep., *Competitive Steel Buildings through Natural Fire Safety Concept.* CEC Agr. 7210-SA/522, Esch/Alzette, Luxembourg, 2000.
[11] International Fire Engineering Design for Steel Structures: *State of the Art.* International Iron and Steel Institute. Brussels, 1993.
[12] Schleich J.-B., Cajot, L.-G.: ECSC Fin. Rep., *Competitive Steel Build. through Natural Fire Safety Concept. Part 4: Statistics.* CEC Agr. 7210-SA/522, Esch/Alzette, Luxembourg, 2000.
[13] Gulvanessian H., Holický M., Cajot L.-G. & Schleich J.-B. *Reliability Analysis of a Steel Beam under Fire Design Situation. Proc. of EUROSTEEL99*, Czech Technical University in Prague, pp. 177-180, 1999.
[14] Gulvanessian H., Holický M., Cajot L.-G. & Schleich J.-B. *Probabilistic Analysis of Fire Safety using Bayesian Causal Network. Proc of ESREL99*, Muenchen, A.A. Balkema, Rotterdam, pp. 725-730, 1999.
[15] Holický M. & Schleich J.-B. *Fire Safety Assessment using Bayesian Causal Network. Proc. of Foresight and Precaution Conf., ESREL 2000,* Edinburgh, A.A. Balkema, Rotterdam, pp. 1301-1306, 2000.
[16] Holický M. & Schleich J.-B. *Estimation of risk under fire design situation. Proc. of Risk Analysis 2000 Conf.,* Bologna, WITpress, Southampton, Boston, to be published 2000.
[17] Ang A.H-S. and W.H. Tang: *Probabilistic Concepts in Engineering Planning and Design. Volume I - Basic principles.* John Wiley, London, 409 p, 1975.
[18] Schneider J.: *Introduction to Safety and Reliability of Structures.* IABSE, Zurich, 138 p, 1997.
[19] Thoft-Christensen P. and M.J. Baker: *Structural Reliability Theory and its Applications.* Springer-Verlag, Berlin, 1982.
[20] Melchers R.E.: *Struct. Reliability Analysis & Pred.* J.Wiley & Sons, Chichester, 437 p, 1999.
[21] Stewart M.G. & Melchers R.E. *Probabilistic Risk Assessment of Engineering System.* Chapman & Hall, London, 274 p, 1997.
[22] Ellingwood B.R. *Probability-Based Structural Design: Prospect for Acceptable Risk Bases. Proc. of Applic. of Statist. and Probability, ICASP 8.* A.A. Balkema,Rotterdam,pp.11-18, 1999.
[23] Hugin System, *Version 5.3 Light (free version).* Hugin Expert A/S, Niels Jernes Vej 10, DK-9220 Aalborg, 1999.

7. Acknowledgement

This research has been partly conducted at the Klokner Institute of the Czech Technical University in Prague, Czech Republic, as a part of the research project CEZ: J04/98/210000029 "Risk Engineering and Reliability of Technical Systems", and partly at ProfilARBED, Esch/Alzette, Luxembourg as a part of the Research Projects "Natural Fire Safety Concept I and II", CEC 7210-SA/522 etc. and CEC 7210-PR060.