Structural Design and Reliability Benchmark Study

Milan HOLICKY
Research fellow
Czech Technical University,
Prague, Czech Republic

Milan Holicky, born 1943, received his Civil Engineering degree from the Czech Technical University (CTU) in Prague in 1965, PhD. from the University of Waterloo in 1971; researcher at the Klokner Institute of CTU.

Joerg SCHNEIDER
Professor emeritus
Swiss Fed. Institute of Technology, Zurich


Summary

This paper compares a number of independent designs and reliability assessments of a simple column supporting a hypothetical car park roof. Designs and reliability assessments are compared considering required dimensions, the failure probability of a given column, and the hours spent on the task.

Possible explanations of the observed scatter are based on the participant's comments and the reporter's assessments of the results.

Keywords: Structures, Safety, Reliability, Design, Benchmark.

1. Introduction

Engineers and risk analysts have been invited to participate in a risk assessment benchmark study, which was conducted prior to the Malta conference of March 2001. The objective was to obtain a sample of independent design or risk assessments, which were analysed prior to the conference and will be discussed during the conference. The main aim is to develop an improved understanding of cultural and human factors in risk assessment processes and a better understanding of the sensitivity of probabilistic results to the assessment process itself.

Three hypothetical exercises were specified well in advance, i.e. a flooding risk exercise, a structural engineering task and a rail tunnel case. The first one did not receive sufficient support, while the latter received a good response. The following summarises the results from the structural exercise.

2. The task

2.1 Introduction

An underground parking garage is to be built somewhere in your country. When necessary assume an altitude of 500 m above sea level. The garage is sheltered by a flat slab supported directly by columns with spacing 5.00 by 7.00 m, which continues over many fields. Its thickness is 0.24 m. Drained earth of 0.50 m that cover the slab serves as a recreational area. Inclement weather (such as rain, snow, etc.) is to be considered. However, no vehicles can access the space.

2.2 Project definition

In order to shorten and facilitate the exercise only an inner column is considered. The benchmark exercise distinguishes three parts, i.e.:

- dimensioning the column according to the codes valid in your country,
- dimensioning mild steel hangers replacing the columns,
- assessing the reliability of the column its dimensions being given.
2.3 Task definition

- Specify the necessary diameter of a circular reinforced concrete column according to the codes of your country. Assume a short column that is not liable to buckle. Consider mild steel and the minimum reinforcement ratio prescribed. Choose the concrete quality that is most prevalent in your country.

- Specify the corresponding dimensions of hangers if these were used instead of columns. How and where the hangers are anchored is not of importance. Keep the column force calculated in the previous step above and calculate the necessary cross section of the hangers according to i) the steel code, ii) the concrete code of your country. In both cases assume ordinary or mild steel.

- Assess the reliability (in terms of the Safety Index $\beta$ according to Hasofer/Lind) of a reinforced concrete column of diameter 0.30 m with 0.6% longitudinal mild steel reinforcement. When necessary, think in probabilities per year. Assume that the concrete cube strength has a mean of 30 N/mm$^2$ and a 2% fractile of 20 N/mm$^2$. The stochastic model of the concrete strength as well as models and parameters of all other variables are up to the choice of the participants and the subject of this benchmark exercise.

3. Participants

The following colleagues (given in alphabetical order) participated in the Benchmark study:

- Michael Havbro Faber, ETH Zurich, Switzerland, together with John Dalsgaard Sorensen, University of Aalborg, Denmark
- Stephan Gollwitzer, RCP GmbH, Barerstrasse 48, Munich, Germany
- Milan Holicky, Klokner Institute, Czech Technical University, Prague, Czech Republic
- Ehrfried Koelz, Risk&Safety Ingenieure in Gemeinschaft, Zurich, Switzerland
- Andrzej S. Nowak, Dept. of Civil and Environm'l Eng., Univ. of Michigan, Ann Arbor, USA
- Markus Petschacher, Consulting Engineer, Am Hügel 4, Feldkirchen, Austria
- Yoshihiro Sasaki, Kajima Corporation, Tokyo, Japan
- Alex Scheiwiller, GRUNER Ltd. Consulting Engineers, Basle, Switzerland
- Jörg Schneider, Schützenstrasse 21, Zollikon, Switzerland
- Mark G. Stewart, University of Newcastle, Newcastle NSW, Australia
- Constantin Trezos, National Technical University of Athens, Athens, Greece
- Dimitri Val, James Cook University, Townsville, QLD, Australia
- Paul H. Waarts, TNO, P.O.Box 49, Rijswijk, The Netherlands

Throughout the paper capitals A, B, C... are used instead of names to distinguish between the results of particular studies. In order to hide the authorship the capitals were associated to the names in a random way.

4. Results

4.1 Design of reinforced concrete column and steel hanger

All participants were asked to fill in an Excel sheet giving loads and actions, design force in the supports, and the diameter of the reinforced concrete column and the steel hanger. An extract of these reply sheets featuring the main input data and the main results, i.e. the diameter of the column and of the hanger are given in Table 1.

All studies used a partial factor method for the design. The only exception is Study L that was applying an "Allowable-Stress" format.

The earth cover density and the live load play an important role. Therefore the table gives the values introduced by the participants. Multiplied by the country’s code specific partial safety factors the
design values of the load in the column and the hanger, respectively, are derived. These values show some scatter which is, however, smaller than the reporters expected.

When addressing the part of the design of the column it is clear that the compressive strength of concrete is of paramount importance. Here, respective codes go their specific ways. Comparison was difficult. In order to gain insight the reporters have chosen to apply all mentioned reduction, conversion, and partial safety factors to the concrete strength thus arriving at a, so-called, design compressive concrete strength, also listed in Table 1.

Furthermore it became obvious that the eccentricity of the force introduced in designing the column or applied bending moments are of importance. Though it was stated in the task that the column is not slender and not liable to buckle, some of the participants introduced eccentricities or bending moments in the design.

In addition to the scatter in the design value of the load, these two factors, compressive strength and eccentricities, result in some scatter of the column diameter ranging from 230 to 380 mm.

### Table 1: Design of column, main input data and results

<table>
<thead>
<tr>
<th>Study</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth cover density in kN/m²</td>
<td>20.0</td>
<td>18.0</td>
<td>18.0</td>
<td>18.0</td>
<td>21.0</td>
<td>21.0</td>
<td>20.0</td>
<td>18.0</td>
<td>20.0</td>
<td>15.0</td>
<td>18.6</td>
<td>18.0</td>
<td>19.1</td>
</tr>
<tr>
<td>Live load in kN/m²</td>
<td>4.0</td>
<td>3.0</td>
<td>5.0</td>
<td>4.0</td>
<td>5.0</td>
<td>5.0</td>
<td>3.0</td>
<td>1.5</td>
<td>5.0</td>
<td>2.5</td>
<td>4.9</td>
<td>5.0</td>
<td>4.8</td>
</tr>
<tr>
<td>Design value N₀ of force in kN</td>
<td>899</td>
<td>814</td>
<td>1038</td>
<td>893</td>
<td>1048</td>
<td>1050</td>
<td>847</td>
<td>672</td>
<td>872</td>
<td>815</td>
<td>704</td>
<td>970</td>
<td>1100</td>
</tr>
<tr>
<td>Design compressive concrete strength N/mm²</td>
<td>13.3</td>
<td>15.3</td>
<td>13.3</td>
<td>13.0</td>
<td>13.3</td>
<td>13.0</td>
<td>12.2</td>
<td>15.2</td>
<td>21.0</td>
<td>11.3</td>
<td>6.4</td>
<td>15.0</td>
<td>15.4</td>
</tr>
<tr>
<td>Diameter of column in mm</td>
<td>290</td>
<td>246</td>
<td>321</td>
<td>358</td>
<td>300</td>
<td>360</td>
<td>280</td>
<td>229</td>
<td>230</td>
<td>300</td>
<td>380</td>
<td>300</td>
<td>300</td>
</tr>
</tbody>
</table>

Concerning the design of the hanger, eccentricities and moments are zero. The diameter clearly should follow the very basic and simple design equation \( A \cdot f_{yd} = N_d \). Here \( N_d \) is the design value of the force in the hanger, \( A \) the area of the hanger and \( f_{yd} \) the design value of the yield strength of the steel. The data shown in Table 2 were reported. Here, again, all reduction factors mentioned were included in what we here call the design yield strength.

### Table 2: Design of hanger, main input data and results

<table>
<thead>
<tr>
<th>Study</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design yield strength of steel in N/mm²</td>
<td>204</td>
<td>270</td>
<td>214</td>
<td>196</td>
<td>214</td>
<td>210</td>
<td>???</td>
<td>286</td>
<td>235</td>
<td>???</td>
<td>???</td>
<td>235</td>
<td>223</td>
</tr>
<tr>
<td>Diameter of hanger in mm</td>
<td>75</td>
<td>62</td>
<td>79</td>
<td>79</td>
<td>79</td>
<td>80</td>
<td>???</td>
<td>55</td>
<td>69</td>
<td>???</td>
<td>???</td>
<td>74</td>
<td>72</td>
</tr>
</tbody>
</table>

The range of diameters between 55 and 80 mm is large due to differences in both the design value of the force in the hanger and of the design yield strength of the steel.

#### 4.2 Structural reliability analysis

The reliability assessment of a column, given the diameter and the concrete strength characteristics, resulted in a quite large scatter. This of course is due to a large variety of definitions of the G-function and of the respective variables. The results reported are shown in Table 3.

### Table 3: Reliability Index and probability of failure

<table>
<thead>
<tr>
<th>Study</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hasofer/Lind reliability index</td>
<td>4.4</td>
<td>4.2</td>
<td>4.1</td>
<td>5.2</td>
<td>4.1</td>
<td>5.9</td>
<td>5.1</td>
<td>5.0</td>
<td>5.5</td>
<td>7.0</td>
<td>4.3</td>
<td>6.1</td>
<td>4.55</td>
</tr>
<tr>
<td>Failure Prob./year, in E-6</td>
<td>0.1</td>
<td>12.0</td>
<td>20</td>
<td>1.0</td>
<td>18</td>
<td>0.00</td>
<td>0.21</td>
<td>0.29</td>
<td>0.02</td>
<td>0.00</td>
<td>8.5</td>
<td>0.00</td>
<td>?</td>
</tr>
</tbody>
</table>

There is some doubt as to whether all the indicated failure probabilities are defined per year, as requested by the Benchmark definition. This part of the study deserves discussion during the Malta Conference.

The participants were asked to extensively comment on the limit state function used and on the distribution type of variables and its parameters. The evaluation of this part of the exercise was quite difficult. The reporters felt the need to go back to the participants asking for clarification and further information. More on this problem area may be found in 6.4.
4.3 Time spent for the exercise

The participants were asked to give an estimate of the number of hours spent working. When judging these numbers it must be considered that the participants were not newcomers to such tasks and most probably could use some specially designed software and templates. The numbers shown in Table 4 were reported.

<table>
<thead>
<tr>
<th>Study</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time used, in man-hours</td>
<td>20</td>
<td>12</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>2</td>
<td>7</td>
<td>5</td>
<td>8</td>
<td>6</td>
<td>24</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4: Time spent to derive solutions

5. Comments of the participants

The participants were asked to comment on their findings. From these comments the following was retained.

- Study A

  The reliability index \( \beta \) of the column may be significantly affected by the eccentricities considered in the study. Generally, three types of eccentricities (first order, additional and second order eccentricity) may be taken into account. In order to account for these eccentricities, a column length of 4 m was assumed in addition to the given information. If all three eccentricities are considered, the reliability index is 3.5. If only the additional eccentricity is considered the reliability index is 5.4. If "a short column that is not liable to buckle" is considered, only the first order and additional eccentricity should be taken into account. In this case the reliability index is 4.4. For reliability calculation the COMREL program and the SORM method were used.

  It would be interesting to extend this study to other materials (i.e. masonry, composed steel-concrete)

- Study B

  It is assumed that the layout of the column is a design outcome. Thus, some variability in the diameter of the column is included, with little influence on reliabilities. The load variable was split into a sustained and an annual extraordinary part. This is however still a simplification of the actual loading process.

  It is stated that calculating probabilities of failure per year is more difficult since most loading models (developed for use in calibration studies) provide probabilistic models for lifetime peak loads, not annual peak loads. The CALREL program was used to calculate FORM reliabilities.

- Study C

  The participant determined the variable snow load by multiplying snow height and snow density. For reliability calculation the VaP program was used. No eccentricity was taken into account:

  - in the design stage, according to the existing concrete code, the accidental eccentricity is taken into account by the minimum percentage of reinforcement,
  - in the reliability analysis the assumption given was followed (short column, not liable to buckle)

  The design of the column (part 1 of the exercise) is designed according to the existing codes of the country, while the reliability analysis (part 3 of the exercise) is done according to the international experience (as it is reflected in some JCSS documents). Therefore, there are some differences in the basic variables in these two parts of the exercise.

- Study D

  The codes of the participant's country leave quite some freedom for the designer, particularly with respect to formally handling "load combinations". Replacing formalities, the concept of hazard scenario is used. Thus, as snow is negatively correlated to live load, the maximum of the one is combined with the minimum of the other. Here, live load is dominant and snow loads are neglected. The hazard scenario labelled "Failure of drainage system" was also studied. It was not dominant as respective loads (earth under water) are smaller than factored earth cover and live loads.

  The eccentricity of loads, according to the code, should be considered even if columns are not slender and buckling is ruled out. The code of our country gives rules related to this problem.
Modelling the eccentricity of the force in the column (or the influence of corresponding bending moments) is certainly not optimal. Time restrictions did not permit going into more depth. The models for earth cover and live loads are engineering guesses made while working with the VaP program, observing the probability densities until they fit to the participant's liking. As the live load model is based on the idea of yearly extremes, the resulting failure probability is per year.

- **Study E**
  It is assumed that failure of the column is due to compression in the inner region of the column. Thus, eccentricity effects are ignored. Therefore, the resistance model becomes very simple. Its model uncertainty should be quite low and is deterministically set to zero. The estimated failure probability may be termed a conditional failure probability “given there is a crowd on the roof in winter”, which is rather unlikely.

- **Study F**
  The participant rightly states that the HL reliability index is strongly dependent on the limit state function and the definitions of the variables. A comparison with other calculations without quoting the assumptions may therefore cause confusion, especially to persons with weak analytical background.

- **Study G**
  The accidental moments caused by imperfections in column straightness and live loads are neglected. This was primarily done due to a lack of data and because it was just a one-level structure. Additionally, the moments were neglected because live loads were a rather small fraction of the total load.
  Most of the time was spent to find data on live loads relevant to the problem. As a result, no data on this type of structure have been found and the live load models used in the study are mainly based on data used for office floor loading (except for the nominal value of live load).

- **Study H**
  The participant states that concrete compression strength is clearly an issue, which deserves some closer consideration in the context of this study. This includes shape and size of the test specimen, the development of strength over time, and combined stress states in the real structure.

- **Study J**
  The participant did not use a G-function. Instead, a geometrically and physically non-linear finite element model describes the mechanical behaviour. Consequently, the column was analysed by means of a probabilistic finite element method. The column is modelled by means of three noded beam elements. The beam elements have 12 integration point in the radial direction, seven in the tangential direction and two in longitudinal direction. The finite element method used is DIANA. The probabilistic method used is directional sampling using an adaptive response surface. The model uncertainty has been calculated by comparing the numerical and test results of concrete columns. The FE- model slightly overestimates the bearing capacity: mean 1.1, st.dev. 0.1.

- **Study K**
  The design of the column and the hanger roughly follows EC2 procedures. A column diameter of 300 mm is recommended although 280 mm is also feasible.
  The β-value results from a SORM analysis using COMREL-TI. The indicated \( p_f = 1.7 \times 10^{-12} \) corresponds to a design life of approximately 50 years. Concrete strength is the variable with the most significant influence. Modelling the height of the earth cover is rather crude and has a big influence. Respective partial safety factors in codes seem insufficient to cover the uncertainties.

- **Study L**
  An allowable stress design method is applied for problem 1 according to the design code valid for underground car-park structures of this country.
  According to the design practice in this country an earthquake load would be accounted for. However, both design and reliability calculations did not account for an earthquake load because the set of loading excluding an earthquake load will be dominant for the given sized structure. It also should be noted that according to the design code snow is not considered as a design load in the assumed area.
Study M

The stochastic models used for resistance as well as for loading are fairly simple. For practising engineers this column will be dimensioned due to constructive rules. Therefore the β-index has a relatively high value of seven, indicating that the column is safe and the design is valid. If the client requires further information, the α-values may be observed and the mechanical and stochastic models might be reconsidered. In that way the concrete strength and also the resistance assumption \( V_{t,R} = A_c \cdot f_c \) may be formulated in a more sophisticated way.

6. Comments by the reporters

6.1 Mistakes and blunders

Besides typing errors there were quite a number of mistakes and blunders in the replies that, had the design been used for execution, have most probably resulted in a structural failure. A few examples:

- One participant used extremely low (obviously erroneous) specific densities of concrete and earth cover for the design of the column.
- Another participant entered the radius of the column instead of the diameter onto the sheet (which certainly would have resulted in a collapse).
- One participant indicated a column cross-section, which was not capable of resisting the value of the design axial load.
- One participant forgot the factor between cylinder an cube strength and the factor between short and long term load.
- One participant took the short term instead of the long term load into the probability analysis.

The above mistakes, of course, were corrected and are no longer contained in the reported results.

6.2 Design formats

All but one of the participants have used partial factor design format. Nevertheless there were some differences in classification of the actions and application of partial safety factors. Most participants used the codes of their respective countries.

6.3 Taking into account eccentricities

In accordance with EC2 three types of eccentricities (first order, additional and second order eccentricity) should be taken into account depending on actual condition. However, as stated in the task definition a short column that is not liable to buckle should be considered. Only the first order moment (due to asymmetric variable loading on the playground) and additional eccentricity due to structural imperfections are applicable in the study. To determine these eccentricities a column height, which was, however, not specified in the task definition, is usually needed.

The first order eccentricity was neglected by almost all of the participants, although experience indicates that depending on the stiffness of the column and the slab its effect on the column reliability may be significant. Various standards (e.g. ACI) provide approximate formulae for determining the bending moment due to asymmetric action in the column supporting a flat slab, which may well be applied in the study case.

The additional eccentricity due to various structural imperfections is specified in EC2 as a fraction of the column length. Although it is of the order of 10 mm it is not negligible in the design and reliability verification. Due to these eccentricities the column is exposed to a combination of the axial force and bending moment.

6.4 Variety of G-functions and respective variables

G-functions used by the participants were quite different. The reporters attempted to draft a G-function that would fit to almost all analyses. This function reads as follows:

\[
G = X_1 \cdot ((\pi/4) \cdot X_2^2 \cdot X_3 \cdot X_4 + X_6 + X_7 \cdot X_8) - X_9 \cdot 5.0 \cdot 7.0 \cdot (X_{10} \cdot X_{11} + X_{12} \cdot X_{13} + X_{14} + X_{15})
\]
Although most of the variables introduced are identifiable from its context they are listed below:

- $X_1$: Model uncertainty variable for resistance
- $X_2$: Diameter of column
- $X_3$: Eccentricity factor
- $X_4$: Conversion factor for concrete strength
- $X_6$: Concrete compressive strength
- $X_7$: Steel area
- $X_8$: Steel yield strength
- $X_9$: Model uncertainty variable for load effects
- $X_{10}$: Density of concrete
- $X_{11}$: Slab thickness
- $X_{12}$: Density of earth cover
- $X_{13}$: Earth cover
- $X_{14}$: Live load
- $X_{15}$: Snow load

The two parts of Table 5 provide an overview and show the distribution type and the moments of the variables. Everyone did not use all of the variables (see 1 or 0, respectively). The first table contains the variables in the resistance part of the G-function, while the second is devoted to the loading part. Two participants endeavoured to split the live load into sustained and short-term parts. One participant introduced the distances between columns as additional variables. The influence of these variabilities, however, was negligible.

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$X_6$</th>
<th>$X_7$</th>
<th>$X_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>N(1.0;0.10)</td>
<td>N(300;5)</td>
<td>Det(0.8)</td>
<td>LN(30;5)</td>
<td>Det(425)</td>
<td>LN(560;30)</td>
</tr>
<tr>
<td>B</td>
<td>N(0.99;0.06)</td>
<td>N(301.5;6.4)</td>
<td>Det(1)</td>
<td>N(0.74;0.044)</td>
<td>LN(24;4.8)</td>
<td>N(424;12.7)</td>
</tr>
<tr>
<td>D</td>
<td>N(1.0;0.05)</td>
<td>N(300;5)</td>
<td>see 9)</td>
<td>LN(0.8;0.05)</td>
<td>LN(30;5)</td>
<td>Det(425)</td>
</tr>
<tr>
<td>E</td>
<td>Det(1)</td>
<td>Det(300)</td>
<td>Det(1)</td>
<td>Det(1)</td>
<td>Det(0.8)</td>
<td>see 11</td>
</tr>
<tr>
<td>F</td>
<td>Det(1)</td>
<td>Det(300)</td>
<td>Det(1)</td>
<td>Det(0.85)</td>
<td>see 10</td>
<td>LN(32;5,49)</td>
</tr>
<tr>
<td>G</td>
<td>N(1.0;0.06)</td>
<td>Det(300)</td>
<td>Det(1)</td>
<td>Det(0.85)</td>
<td>LN(25;4.8)</td>
<td>Det(425)</td>
</tr>
<tr>
<td>H</td>
<td>LN(1.0;0.05)</td>
<td>Det(300)</td>
<td>Det(1)</td>
<td>Det(0.91)</td>
<td>LN(25;4.8)</td>
<td>Det(425)</td>
</tr>
<tr>
<td>J</td>
<td>N(1.0;1.0)</td>
<td>N(300;4)</td>
<td>Det(1)</td>
<td>N(0.72;0.04)</td>
<td>Det(30;5.7)</td>
<td>Det(425)</td>
</tr>
<tr>
<td>K</td>
<td>Det(1)</td>
<td>Det(300)</td>
<td>Det(0.85)</td>
<td>Det(1)</td>
<td>LN(30;5.7)</td>
<td>Det(425)</td>
</tr>
<tr>
<td>L</td>
<td>Det(1)</td>
<td>N(300;15)</td>
<td>Det(1)</td>
<td>Det(1)</td>
<td>N(30;4.9)</td>
<td>0 –</td>
</tr>
<tr>
<td>M</td>
<td>LN(1.0;0.15)</td>
<td>N(300;5)</td>
<td>Det(1)</td>
<td>Det(1)</td>
<td>Det(1)</td>
<td>LN(30;4.0)</td>
</tr>
<tr>
<td>N</td>
<td>N(2275;320)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_9$</th>
<th>$X_{10}$</th>
<th>$X_{11}$</th>
<th>$X_{12}$</th>
<th>$X_{13}$</th>
<th>$X_{14}$</th>
<th>$X_{15}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>N(1.1;0.11)</td>
<td>N(25.0;2.0)</td>
<td>N(20;2)</td>
<td>N(0.5;0.1)</td>
<td>Gu(1.1;0.56)</td>
<td>???</td>
</tr>
<tr>
<td>B</td>
<td>Det(1)</td>
<td>see 4)</td>
<td>see 6)</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Det(1)</td>
<td>N(25;0.1)</td>
<td>Det(240)</td>
<td>N(23;0;1.15)</td>
<td>N(0.5;0.05)</td>
<td>Gu(2.5;0.3)</td>
</tr>
<tr>
<td>D</td>
<td>N(1.0;0.05)</td>
<td>N(24.5;0.7)</td>
<td>N(240;6)</td>
<td>N(19;1.2)</td>
<td>LN(0.5;0.08)</td>
<td>Gu(3.0;0.6)</td>
</tr>
<tr>
<td>E</td>
<td>N(1.0;0.08)</td>
<td>N(25;0;1.0)</td>
<td>N(240;6)</td>
<td>N(21;1.1)</td>
<td>N(0.5;0.05)</td>
<td>LN(1.24;1.11)</td>
</tr>
<tr>
<td>F</td>
<td>Det(1)</td>
<td>LN(25;0.10)</td>
<td>Det(240)</td>
<td>LN(21;0.2;1)</td>
<td>LN(0.5;0.03)</td>
<td>Gu(3.0;1.2)</td>
</tr>
<tr>
<td>G</td>
<td>N(1.05;0.105)</td>
<td>see 5)</td>
<td>see 7)</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Det(1)</td>
<td>N(6.0;0.36)</td>
<td>incl’d in X_{10}</td>
<td>N(9;0;0.9)</td>
<td>incl’d in X_{12}</td>
<td>Gu(0.99;0.2)</td>
</tr>
<tr>
<td>J</td>
<td>Det(1)</td>
<td>N(6.0;0.43)</td>
<td>incl’d in X_{10}</td>
<td>N(21;1.5)</td>
<td>incl’d in X_{12}</td>
<td>Gu(2.5;1.0)</td>
</tr>
<tr>
<td>K</td>
<td>Det(1)</td>
<td>N(23;1;1.16)</td>
<td>N(240;2.5)</td>
<td>N(15;0;1.5)</td>
<td>N(0.5;0.1)</td>
<td>Gu(1.52;0.38)</td>
</tr>
<tr>
<td>L</td>
<td>Det(1)</td>
<td>N(24.5;0.12)</td>
<td>N(240;12)</td>
<td>N(18;6;0.93)</td>
<td>N(0.5;0.025)</td>
<td>N(65.0;2)</td>
</tr>
<tr>
<td>M</td>
<td>Det(1)</td>
<td>N(6;0;0.3)</td>
<td>N(9;0;0.6)</td>
<td>Gu(5;0;0.8)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>Det(1)</td>
<td>N(15;4;1.54)</td>
<td>N(47;7;0.71)</td>
<td>N(0.97;0.17)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Definition of variables

Notes: 1) Eccentricities were considered in a different way.
2) Beta(460;45;372;703)
3) Snow density N(1.25;0.36) multiplied by snow height Gu(0.45;0.25)
4) N(1.05 Nom;0.1 Nom) for both dead load and earth load, in total N(15.8;1.5)
5) N(1.05 Nom;0.1 Nom) for both dead load and earth load, in total N(16.5;1.7)
6) Sustained live load Gamma(31.5;18.9) + transient live load Gamma(20.0;13.2)
7) Sustained live load Gamma(0.9;0.54) + transient live load Gamma(0.57;0.57)
8) Live load scenario suggested to exclude snow
9) (1-2.4 Ex/D) with Ex = sExp(0.004;0.004)
10) Eccentricity introduced in model with N(0.006;0.005)
11) replaced by $X_6 \cdot e^{0.96 \cdot Y}$, with $X_6 = LN(33;5)$ and $Y = LN(1.0;0.06)$
There is some interest of course in the relative weight of a variable on the result. This weight is given by the $\alpha$-values which are part of the results of a FORM or SORM analysis. These values, as communicated by the participants, are given in Table 6. The two largest values in each row are printed in bold.

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$X_5$</th>
<th>$X_6$</th>
<th>$X_7$</th>
<th>$X_8$</th>
<th>$X_9$</th>
<th>$X_{10}$</th>
<th>$X_{11}$</th>
<th>$X_{12}$</th>
<th>$X_{13}$</th>
<th>$X_{14}$</th>
<th>$X_{15}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.339</td>
<td>0.089</td>
<td>0.545</td>
<td>0</td>
<td>0.222</td>
<td>0</td>
<td>0.043</td>
<td>0.268</td>
<td>0.030</td>
<td>0.031</td>
<td>0.068</td>
<td>0.127</td>
<td>0.432</td>
<td>0.018</td>
</tr>
<tr>
<td>B</td>
<td>0.335</td>
<td>0.166</td>
<td>0</td>
<td>0.247</td>
<td>0.765</td>
<td>0.036</td>
<td>0.110</td>
<td>0</td>
<td>0.395</td>
<td>0</td>
<td>0.209</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.985</td>
<td>0</td>
<td>0.023</td>
<td>0</td>
<td>0.028</td>
<td>0</td>
<td>0.071</td>
<td>0.137</td>
<td>0.057</td>
<td>0.040</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0.233</td>
<td>0.164</td>
<td>0.509</td>
<td>0.223</td>
<td>0.591</td>
<td>0</td>
<td>0.024</td>
<td>0.208</td>
<td>0.018</td>
<td>0.028</td>
<td>0.157</td>
<td>0.417</td>
<td>0.130</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.438</td>
<td>0</td>
<td>0.024</td>
<td>0.172</td>
<td>0.019</td>
<td>0.010</td>
<td>0.044</td>
<td>0.082</td>
<td>0.877</td>
<td>0.023</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.646</td>
<td>0</td>
<td>0.106</td>
<td>0</td>
<td>0.041</td>
<td>0</td>
<td>0.208</td>
<td>0.125</td>
<td>0.713</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>0.360</td>
<td>0</td>
<td>0</td>
<td>0.792</td>
<td>0</td>
<td>0.091</td>
<td>0.397</td>
<td>all zero, variability included in $X_9$</td>
<td>0.277</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>0.324</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.875</td>
<td>0</td>
<td>0.031</td>
<td>0</td>
<td>0.128</td>
<td>0</td>
<td>0.320</td>
<td>0</td>
<td>0.072</td>
<td>0.063</td>
</tr>
<tr>
<td>J</td>
<td>0</td>
<td>0</td>
<td>0.420</td>
<td>0.050</td>
<td>0.700</td>
<td>0</td>
<td>0.240</td>
<td>0.060</td>
<td>0</td>
<td>0.490</td>
<td>0</td>
<td>0.170</td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.815</td>
<td>0</td>
<td>0.113</td>
<td>0</td>
<td>0.070</td>
<td>0.015</td>
<td>0.306</td>
<td>0.454</td>
<td>0.114</td>
<td>0.070</td>
</tr>
<tr>
<td>L</td>
<td>0</td>
<td>0.203</td>
<td>0</td>
<td>0</td>
<td>0.975</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.035</td>
<td>0.004</td>
<td>0.056</td>
<td>0.056</td>
<td>0.022</td>
<td>0</td>
</tr>
<tr>
<td>M</td>
<td>0.533</td>
<td>0.182</td>
<td>0</td>
<td>0</td>
<td>0.709</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.069</td>
<td>0.138</td>
<td>0.395</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.077</td>
<td>0.018</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: $\alpha$-values

From this table it can be seen that there is no common ground for a reply to the question concerning the specification of the most important variables in the G-function. It appears that the concrete strength plays a dominant role.

### 6.5 Effective concrete strength

The concrete strength, however, was well defined in the benchmark task. Quote: *Assume that the concrete cube strength has a mean of 30 N/mm$^2$ and a 2% fractile of 20 N/mm$^2$*. The participants translated this information in various ways into moments of which they thought would represent the effective strength of the concrete relevant to the carrying capacity of the column. It appears that the concrete strength was modelled by multiplying $X_4$ and $X_6$. The moments of these two variables and of their product are given in Table 7.

<table>
<thead>
<tr>
<th>$X_4$</th>
<th>$X_6$</th>
<th>$F_{\text{eff}} = X_4 X_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Det(0.8)</td>
<td>LN(30;5) LN(24.0;4.0)</td>
</tr>
<tr>
<td>B</td>
<td>N(0.70;0.044)</td>
<td>LN(24;4.8) LN(16;3.5)</td>
</tr>
<tr>
<td>C</td>
<td>Det(1)</td>
<td>N(25.5;4.15) LN(25.5;4.2)</td>
</tr>
<tr>
<td>D</td>
<td>LN(0.8;0.05)</td>
<td>LN(30;5) LN(24;0;4.3)</td>
</tr>
<tr>
<td>E</td>
<td>Det(1)</td>
<td>LN(23.3) LN(23.0;5.0)</td>
</tr>
<tr>
<td>F</td>
<td>LN(0.7;0.042)</td>
<td>LN(33;5) LN(22.9;3.6)</td>
</tr>
<tr>
<td>G</td>
<td>Det(0.85)</td>
<td>LN(25;4.8) LN(21;3.4;1)</td>
</tr>
<tr>
<td>H</td>
<td>Det(0.91)</td>
<td>LN(32.5;4.9) LN(29.6;4.5)</td>
</tr>
<tr>
<td>J</td>
<td>N(0.72;0.04)</td>
<td>LN(30;5.7) LN(21.6;4.3)</td>
</tr>
<tr>
<td>K</td>
<td>Det(0.85)</td>
<td>LN(30;5.7) LN(25.0;4.9)</td>
</tr>
<tr>
<td>L</td>
<td>Det(1)</td>
<td>N(30;4.9) N(30.0;4.9)</td>
</tr>
<tr>
<td>M</td>
<td>Det(1)</td>
<td>LN(30;4.0) LN(30.0;4.0)</td>
</tr>
<tr>
<td>N</td>
<td>Det(1)</td>
<td>N(22.8;3.2) N(22.8;3.2)</td>
</tr>
</tbody>
</table>

Table 7: Effective concrete compressive strength

This table indicates that there is a problem: modelling relevant concrete compressive strength is a task that needs better guidance.

### 7. Comments by the participants

The participants were given the opportunity to see the draft of this summary and to add comments, particularly in view of oral discussions during the Malta conference. The design of the column and the reliability assessment as such and all numbers reported above could not be changed in this additional round. The following remarks were retained from this second round.
• Study A
The Benchmark study proves to be an extremely useful mirror of the current situation in the field of structural reliability. Information and data submitted by the participants confirm the fact that research and development in probabilistic methods of structural reliability are far in advance of practical application and design practice. It appears that similar benchmark studies covering different materials and various structural elements are needed.

• Study B
Many codes of practice base their strength prediction equations on cylinder strengths, not cube strengths. Yet it is well known that for the same concrete cylinder strengths will be lower than cube strengths. This could account for some of the reported variability of results.

• Study C
It is evident that concrete strength is the most important variable. This is not only due to the large variability of the concrete but, mainly, to the fact that the column is under-reinforced (the minimum percentage of the reinforcement is 0.8 % to 1.0 % in most codes) and thus the main resisting variable is the concrete.

• Study D
Most intriguing was the inclusion of the eccentricity effect into the G-function. The resistance model uncertainty variable could also cover this effect. Obviously, the weight of these two variables was overestimated.

• Study E
The assessment of reliability usually implies dealing with a lack of knowledge (eg. what are the relevant scenarios or, if two different models are available for describing the behaviour of a structure which one should be chosen?). Usually a decision is made against or in favour of a certain scenario or a certain model. This decision can be made on an individual basis or by a group of experts talking together. But, what is the impact of such decisions on the estimated reliability, which is in fact conditioned on such decisions? Would it be helpful to introduce statements of belief like "based on our knowledge we believe that model A is the right one with an 80 % likelihood while model B deserves just 20 %", and to propagate this uncertainty throughout the reliability assessment. Another interesting question: how much does our expectation on the results influence the modelling process? Why do the results of the benchmark study lie in a relatively narrow band, whereas the underlying assumptions differ widely?

• Study F
It is quite remarkable how large the range of the results are. One of the reasons is certainly the choice of the determinant hazard scenario. In fact, the handling of different hazard scenarios and of different time dependent loads in the scope of a realibility analysis are essential topics needing further discussion.

The differences in the concrete models show the need for a probabilistic model code in order to provide a sound basis for the integration of probabilistic analysis in the daily work and for the comparison of different studies.

• Study H
As stated prior to seeing the results, the issue of the modelling of the concrete compression strength is a central one. No eccentricities were considered in the reliability analysis of the concrete column as it was assumed that this case was supposed to be representative of pure compression failure of concrete structural members, as in code calibration. The choice of the characteristic values of materials and loads will govern the diameter. The ratio between the resulting/effective material factor to the resulting/effective load factor could possibly better illustrate the differences in (deterministic) safety factors achieved by applying different codes.

The result of the exercise appears to be an all-together positive one. It delivered rather consistent results when the effect of different characteristic loads and material strengths are accounted for.
• Study J
The reliability index is higher than the required reliability index (according to the informative annex of the Eurocode $\beta = 3.6$). The high reliability index is due to: a) the dimensions of the column are larger than required by the code, b) the model code uses a very conservative model for an average column, c) the model code uses a quite large eccentricity or bending moment in the column. Without this bending moment, the column is not sensitive to buckling. In this case there is only a low eccentricity. The model code is even more conservative then.

• Study K
The range of results is remarkable. It is due to the quite different assumptions in the stochastic model, especially for the load variables. Several participants, including the author of this study, neglected the eccentricity which clearly is an error, see section 6.3. If eccentricity is included (and the somewhat underestimated density of the earth cover is corrected) one arrives at a reliability index of about 4.5 which seems reasonable. It is unclear why the studies D and K arrive at rather large reliability indices despite the use of eccentricity by their authors.

The benchmark clearly shows that a probabilistic model code providing stochastic models for the most relevant load and resistance quantities is needed to arrive at comparable results.

• Study M
In this study fairly simple mechanical as well as stochastic models have been used. The resulting high $\beta$-values reflect 1) the model assumptions and 2) the constructive aspects in designing such a column. The theoretical diameter would be smaller, but due to unexpected eccentricity (which is neglectable if the slenderness is small) and practical consideration concerning concreting a larger diameter might taken.

Table 6 shows that the concrete strength is one of the most important basic variables. The results for the second most important variable are divergent and reflect the fine tuning of the various limit state functions with one major difference, either it is the eccentricity influence or it is one load component. This seems for me the starting point when reviewing my own assumptions and my formulation of limit state.

• Study N
There is a lot of subjective judgment involved in this simple exercise. The major load component is dead load. I tried to determine what is the minimum diameter of the column and I found that it is not specified in the Code of our country. I asked my colleagues, nobody knew for sure. I ended up using what we thought the limit value. The minimum number of bars for a circular column is 6. The minimum bar size is about 10 mm.

8. Acknowledgements
It needs some courage to join a Benchmark exercise because of the professional exposure, of possibly spreading consulting know-how to others, and with regard to the time to be invested into the study. Therefore, special thanks go to all that decided to join the exercise for their openness, frankness, and endeavour.